The Faulty Assumptions of the Expanding-Universe Model vs. the Simple and Consistent Principles of a Flat-Universe Model

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Abstract The standard model of expanding universe is based on the theory of general relativity (GR) which assumes that spacetime is curved. The reason of curved spacetime was given by Einstein that locally there is common acceleration for all test particles so that gravity is cancelled locally. This is called equivalence principle. The present paper shows that it is not true for Schwarzschild solution (static gravity of pure spatial inhomogeneity). The paper also presents isotropic but temporally inhomogeneous gravity. Freely falling particles do not share common accelerating direction locally which indicates that gravity can not be cancelled too. Realistic gravity is non-static which is the case in between. This indicates that the assumption of curved spacetime is a fundamental mistake. Therefore, a correct gravitational theory or a model of the universe must be based on the absolute flat background spacetime. The existence of such absolute spacetime is shown to be true from the following three basic principles about the universe: (1) the density of large-scale mass distribution of the universe varies with time (corresponding to an isotropic but temporally inhomogeneous gravitational field); (2) the gravity is described by a Lagrangian which is the generalization to the proper distance of special relativity (the metric form of GR); (3) Hubble law is approximately true. These lead to varying light speed and give account of galactic redshifts and 'accelerating expansion'. Therefore, the assumption of big bang and expansion is incorrect. My model is ready for further test.

keywords: Special Relativity – General Relativity – Gravitational Theory - Hubble Law – Cosmology

1 Introduction and Einstein's Mistake

Physical science describes spatial and temporal relative motions due to physical interactions. People found four fundamental interactions and the gravitational one is believed to be the cause of global structure and evolution of the universe. Gravity is the longest known yet the least understood. In Newtonian theory, gravity is considered a static force, a stereotype of "action-at-a-distance" which implies infinite velocity of propagation. One of the fundamental theories discovered in the twentieth century is the theory of special relativity which requires any force be transmitted with velocity less than or equal to c, the light speed. The theory assumes flat background spacetime of Minkowski metric and requires a covariant four-dimensional formulation for all laws of mechanics. For example (Goldstein, 1950), the Lagrangian of any force should be an invariant property of the corresponding system only, independent of the particular coordinate system used, and we expect it to be a world scalar, invariant under all Lorentz transformations. Specifically, we should not treat time t as a parameter entirely distinct from the spatial coordinates. An invariant parameter p must be chosen and the common velocity $\dot{x}^i$ must be replaced by the generalized velocity, dx^α/dp. The correct expression of Lagrangian in four-dimensional language should be

$$L(x^\alpha, \frac{dx^\alpha}{dp}, p) \tag{1}$$

and its action is

$$I = \int L(x^\alpha, \frac{dx^\alpha}{dp}, p) dp \tag{2}$$

with both I and L as world scalars. Such covariant Lagrangian on flat spacetime leads to the Hamiltonian of the system by a Legendre transformation. The common procedure, $P^i \propto \nabla$, produces the quantization of the system. All interactions except gravity have been united successfully by this Lagrangian formulation of Lorentz covariance. The formulation is quantum field theory.

Gravity has been given a theory, general relativity (GR), which assumes curved spacetime. However, the calculation of GR works much better without the assumption (He, 2005c and 2006). I call this simplified explanation of GR by "flat spacetime general relativity" (FGR) whose calculation is similar to GR except that the background Minkowski spacetime is flat and gravity has no geometric meaning. Therefore, FGR can be presented as follows in the fashion of classical mechanics of Lorentz covariance.

Particle gravitational dynamics is based on the following important type of Lagrangian,

$$\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L(x^0, x^i, \grave{x}^0, \grave{x}^i) = \frac{1}{2} g_{\alpha\beta} \grave{x}^\alpha \grave{x}^\beta. \tag{3}$$

where

$$\grave{x}^i = \frac{dx^i}{dp}, \text{ etc.,} \tag{4}$$

p is the curve parameter, ($x^0 = ct = \tilde{t}, x^1 = x, x^2 = y, x^3 = z$) is the Cartesian coordinates on the flat Minkowski spacetime, c is light speed, and $g_{\alpha\beta}$ is a tensor which satisfies general covariance with respect to all curvilinear coordinate transformations. Note that, from now on, any letter with grave accent (e. g., $\grave{x}$) denotes the derivative of the quantity (e. g. x) with the curve parameter p. This is to be distinguished from the dot accent (e. g., $\dot{x}$) which is the derivative of the quantity (e. g. x) with time t, the common notation. The Lagrangian is the generalization to the proper distance $d\bar{s}^2 = d\tilde{t}^2 - (dx^2 + dy^2 + dz^2)$ of flat Minkowski spacetime and $\bar{s}$ is required to be a scalar, the principle of special relativity. Some mathematicians can prove the assertion that p be always proportional to $\bar{s}$. Therefore, p and accordingly the Lagrangian L are scalars. We can find several examples supporting the assertion in the following Sections. Therefore, I have the Lorentz covariant formulation of gravitational interaction which people sought for over eighty years. I note that L (or $\bar{s}$) is invariant (scalar) with respect to all curvilinear coordinate transformations (including Lorentz transformations) and its expression is covariant with respect to all curvilinear coordinate transformations (including Lorentz transformations). I call the specific type of Lagrangian (3) by homogeneous Lagrangian because it is a homogeneous order-two form of the components of the generalized particle velocity. The well known example of homogeneous Lagrangian is the Schwarzschild metric form which describes the gravity of a single point-mass M (e. g., the sun),

$$\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L = \frac{1}{2}(1 - 2r_g/r)c^2\grave{t}^2 - \left(\frac{1}{2}\grave{r}^2/(1 - 2r_g/r) + \frac{1}{2}r^2(\grave{\theta}^2 + \sin^2\theta\grave{\phi}^2)\right) \tag{5}$$

where $r_g = GM/c^2$ and G is the gravitational constant. The corresponding Lagrange's equation gives the particle (a planet or a light wave) dynamics of the solar system which passed the tests of solar observations with great success.

Because it is a Lorentz covariant formulation, the homogeneous Lagrangian has specific meaning which the Lagrangian of Newtonian mechanics has not. Newtonian dynamics is defined on the inertial frame which is a flat Euclidean spacetime on which the distance s between two events satisfies the Pythagoras theorem

$$\frac{1}{2}ds^2 = \frac{1}{2}(d\tilde{t}^2 + dx^2 + dy^2 + dz^2). \tag{6}$$

We can transform the Cartesian coordinates into whatever curvilinear coordinates but the distance (not the form) remains the same. This is called the distance invariance of Euclidean space. However, special relativity is the correct theory of spacetime. It is the following distance (called proper distance) as well as the form that are invariant with respect to all Lorentz coordinate transformations. However, only the proper distance (not the form) is invariant with respect to all curvilinear coordinate transformations,

$$\frac{1}{2}d\bar{s}^2 = L = \frac{1}{2}(d\tilde{t}^2 - (dx^2 + dy^2 + dz^2)). \tag{7}$$

This form involves minus sign and is indefinite (positive, negative, or zero). For example, if we assume the two events are the two ticks from a static clock then $dx = dy = dz = 0$ and $d\bar{s} = d\tilde{t}$. The two events have causal connection and their proper distance in this case is just the temporal distance between the two events. The other extreme case is to consider the length of a rod. This time $d\tilde{t} = 0$. The two events correspond to the two end points of the rod at a simultaneous time and have no causal connection. The corresponding proper distance is negative which equals the minus length of the rod. There is the third case when $d\bar{s} = 0$. The two events are causal but their connection can only be realized by a light signal which travels with maximum speed. We are interested in the world lines of massive particles or light waves. Therefore, $d\bar{s}^2 \geq 0$ generally. If $d\bar{s}^2 > 0$ then $\bar{s}$ can always be explained as the temporal distance between two ticks of a static clock as explained above.

In an inertial frame which presents gravity, all physical processes, e. g., the ticks of a static clock, are affected by the gravitational interaction. Therefore, the partial meaning of the Lagrangian (3) is the ratio of the period $d\tilde{t}$ between two ticks of a static clock in the frame to the period $d\bar{s}$ if the clock had not experienced gravity and had been static:

$$\frac{d\bar{s}}{d\tilde{t}} = \sqrt{g_{00}}. \tag{8}$$

However, we can not observe the time dilation factor g_{00} because, in the same inertial frame, the gravitational field affects our time standards in exactly the same way as it affects the clock being studied. However, we can compare the time dilation factors at two different points. For example, suppose that at point 1 (the Milky Way galaxy) we observe the light coming from a particular atomic transition at point 2 (the remote galaxy). If the galaxies are at rest with respect to the absolute inertial frame of the universe, then the time taken for a wave crest to travel from 2 to 1 will be a constant, given by the integral of $d\tilde{t}$ over the path, and therefore the time between the arrival at point 1 of successive crests will equal the time $d\tilde{t}_2$ between their departure at point 2, given by (8) as

$$d\tilde{t}_2 = \frac{d\bar{s}}{\sqrt{g_{00}(x_2)}} \tag{9}$$

where $g_{\alpha\beta}$ describes the isotropic but temporally inhomogeneous gravity of the universe. If the same atomic transition occurs at point 1, then the time between crests of the light waves to be observed at point 1 will be

$$d\tilde{t}_1 = \frac{d\bar{s}}{\sqrt{g_{00}(x_1)}}. \tag{10}$$

Hence, for a given atomic transition, the ratio of the frequency (observed at point 1) of the light from point 2 to that of the light from point 1 will be

$$\frac{\nu_2}{\nu_1} = \left(\frac{g_{00}(x_2)}{g_{00}(x_1)}\right)^{1/2}. \tag{11}$$

This is exactly my explanation of galactic redshift (Section 3):

$$z = \frac{\nu_1}{\nu_2} - 1 = \frac{\sqrt{g_{00}(x_1)}}{\sqrt{g_{00}(x_2)}} - 1. \tag{12}$$

Excuse me for my English difficulty, the current paragraph are mostly taken from Weinberg (1972).

About my FGR I emphasize again that the Lagrangian (3) has no geometric meaning and its partial interpretation is the ratio of the period $d\tilde{t}$ between two ticks of a static clock in the inertial frame of flat spacetime to the period $d\bar{s}$ if the static clock had not experienced gravity. Gravity affects the speed and path of light propagation (i. e., extremizing the Lagrangian, the effective distance $\bar{s}$) in the similar way as the dielectric medium curves the propagation of light waves (extremizing refractive index n). The scalar $\bar{s}$ has no geometric meaning. It would be the proper distance of spacetime if gravity was not present in the inertial frame. From now on, gravity is called gravitational medium on flat inertial spacetime. The tensor $g_{\alpha\beta}$ is called effective metric and $\bar{s}$ effective distance. Similarly we have effective curvature tensor, etc.

However, Einstein insisted on that the quantity $d\bar{s}$ in (3) is the proper distance of spacetime even when gravity is present. Therefore, the global spacetime manifold has to be curved. The reason of curved spacetime was given by Einstein that locally there is common acceleration for all test particles so that gravity is cancelled locally. This is called equivalence principle. I show that it is not true for Schwarzschild solution (static gravity of pure spatial inhomogeneity, see the Appendix section). I also present isotropic but temporally inhomogeneous gravity. Freely falling particles do not share common accelerating direction locally which indicates that gravity can not be cancelled locally too. **Any isotropic gravitational field on flat spacetime is similar to fluid pressure which, at each spatial position, exerts the same amount of force in all directions. Therefore, static frame in the field remains static and the isotropic gravitational force is not completely embodied by particle's motion. Einstein's equivalence principle is nothing but the cancellation of gravitational force by the particle's motion under the force, which is not true to isotropic gravitational force.**

This non-equivalence holds to isotropic gravity (e. g., the large-scale universe) of pure temporal inhomogeneity. Realistic gravity is non-static, which is the case between the static gravity of pure spatial inhomogeneity and the isotropic gravity of pure temporal inhomogeneity. This indicates that the assumption of curved spacetime is a fundamental mistake. Therefore, a correct theory of gravity or a model of the universe must be based on the absolute flat background spacetime. The existence of such absolute spacetime is shown to be true from the following three basic principles about the universe (Section 3): (1) the density of large-scale mass distribution of the universe varies with time (corresponding to an isotropic but temporally inhomogeneous gravitational field); (2) the gravity is described by a Lagrangian which is the generalization to the proper distance of special relativity (the metric form of GR); (3) Hubble law is approximately true. These lead to varying light speed and give account of galactic redshifts and Hubble law. Therefore, the assumptions of universal equivalence principle, big bang, and expansion are incorrect. I give a partial list of the mistakes which GR has made. The equivalence principle is on top of the list:

Mk (1) (i. e., Mistake (1)): Gravity is cancelled locally by assuming that there is local common acceleration for all test particles (Einstein's equivalence principle). The following Section 3 and the Appendix section give the examples of gravity which have no such cancellation.

Mk (2): There is no absolute inertial frame in the universe. Mach's principle, however, suggests the absolute frame, which states that your hands rise automatically toward your shoulder level when you rotate your body with respective to the background universe. The present paper (Section 3) justifies the principle by showing that the absolute inertial frame (i. e., the absolute flat background spacetime) does come naturally from a set of three simple and consistent principles.

Mk (3): Homogeneous gravity can be geometrized. The beloved example for Einstein to explain his GR is the homogeneous gravitational field. A small part of gravitational field near the earth surface can be considered homogeneous and Einstein's favorite experiment is to sit inside a freely-falling elevator. Inside the elevator, it is true that no gravity can be detected. However, the gravity can not be geometrized. That is, both the elevator frame and the earth observer can not say they have a curved spacetime. The person inside the elevator indicates a flat spacetime because he detects no gravity. So does the earth observer because the two frames are connected by a global coordinate transformation. Global coordinate transformations can not change the flatness of spacetime. Consider an example: sphere surface. You can not transform its metric form into the Pythagoras theorem by whatever global coordinate transformation.

Mk (4): The assumption of curved spacetime passed solar test or other tests. If we assume spacetime is curved then all t, x, y, z in (3) have no direct meaning of spatial and temporal distances and are merely curvilinear coordinates. To calculate spatial and temporal distances or angles between two lines (e. g., the deflection of light by the sun), the coefficients $g_{\alpha\beta}$ must be employed. That is, we need follow the rule of curved geometry to calculate distances and angles on curved 4-dimensional surface (the spacetime manifold). If we do follow the rule then the general relativistic prediction of the deflection of light at the limb of the sun is 1.65 arcseconds (Crothers, 2005). However, all GR text books give the value 1.75 arcseconds which approaches the observed value. These books ignore the rule of differential geometry and treat the coordinates r, θ to be real distance and angle, which is possible only when the spacetime is flat. Therefore, the authors of these books are actually assuming that the spacetime is flat and θ is the angle on the spacetime!

Along with these mistakes, GR is not successful. Different forms of the tensor $g_{\alpha\beta}$ describe different kinds of gravitational fields. GR requires its geometrical meaning and therefore its connection to the curvature of spacetime. Einstein equation connects the curvature to energy momentum tensor. The exact solutions of the equation for $g_{\alpha\beta}$ are very few. Even the few solutions have difficulty in the explanation of astronomic observation and people resort to dark matter. My FGR requires no geometric connection and we are free to study whatever kind of $g_{\alpha\beta}$.

Gravitational field $g_{\alpha\beta}$ describes the inhomogeneity of spacetime. Any Lagrangian whose coefficients $g_{\alpha\beta}$ are not functions of time describes spatial inhomogeneity. Static homogeneous gravity and the gravity of a single point-mass (Schwarzschild metric) are such examples. However, only the following Lagrangian describes an isotropic but temporally inhomogeneous gravity,

$$\begin{aligned} \frac{1}{2}\grave{s}^2 &= L(x^0, x^i, \grave{x}^0, \grave{x}^i) \\ &= \frac{1}{2}B(\tilde{t})(\grave{x}^0)^2 - \frac{1}{2}A(\tilde{t})\sum_{i=1}^{3}(\grave{x}^i)^2 \\ &= \frac{1}{2}g_{\alpha\beta}\frac{dx^\alpha}{dp}\frac{dx^\beta}{dp} \end{aligned} \tag{13}$$

where

$$g_{00} = B(\tilde{t})(>0), g_{11} = g_{22} = g_{33} = -A(\tilde{t})(<0), g_{\alpha\beta} = 0(\alpha \neq \beta). \tag{14}$$

With further determination of the coefficients $A(\tilde{t})(>0)$ and $B(\tilde{t})(>0)$, the Lagrangian can be used to model the universe (see Section 3). In the same Section we will find out that there is no local common acceleration which cancels gravity. Instead, all freely-falling particles decelerate and finally are static with respect to the absolute inertial frame of the universe. To be prepared with those calculations, I have the whole Section 2 devoted to the study of static homogeneous gravity. This is Einstein's favorite example which, however, denies geometrization, contrary to his expectation unfortunately. The last section is Appendix which indicates that Einstein's equivalence principle fails to Schwarzschild solution.

2 Varying Light Speed in Static Homogeneous Gravitational Field

(i) The lagrangian for static homogeneous gravitational field. Consider static homogeneous gravitational field, $\tilde{g} = g/c^2$, in the positive direction of x-axis. For simplicity, I drop off the coordinates y, z when dealing with homogeneous gravity. The coefficients $g_{\alpha\beta}$ of the Lagrangian can be chosen to be the functions of spatial coordinate x only, no time coordinate involved (He, 2005c), which is the square coordinate-rescale cancellation of gravity. Here I present the Lagrangian whose coefficients depend on time only, which is simply obtained by employing the freely-falling coordinate transformation,

$$\begin{aligned} \tfrac{1}{2}\left(\tfrac{d\bar{s}}{dp}\right)^2 &= L = (1-\tilde{g}^2\tilde{t}^2)\left(\tfrac{d\tilde{t}}{dp}\right)^2 + 2\tilde{g}\tilde{t}\tfrac{d\tilde{t}}{dp}\tfrac{dx}{dp} - \left(\tfrac{dx}{dp}\right)^2 \\ &= g_{\alpha\beta}\tfrac{dx}{dp}\tfrac{dx}{dp} \end{aligned} \tag{15}$$

where $\tilde{g} = g/c^2(>0)$ is a constant and

$$g_{00} = 1-\tilde{g}^2\tilde{t}^2, g_{01} = g_{10} = \tilde{g}\tilde{t}, g_{11} = -1. \tag{16}$$

Because static homogeneous gravity describes spatial inhomogeneity (anisotropic space which has the specific direction of x-axis), the above Lagrangian with time-dependent coefficients is not an appropriate description and we do have the difficulty in finding compatible initial values of x and $\tilde{t}$, as indicated in the following.

(ii) Failure of the geometrization of static homogeneous gravity. The static homogeneous gravitational field (15) can be canceled by a global space-time coordinate transformation called the freely-falling coordinate transformation,

$$\begin{aligned} \xi &= x - \tfrac{1}{2}gt^2 = x - \tfrac{1}{2}\tilde{g}\tilde{t}^2, \\ \tau &= t. \end{aligned} \tag{17}$$

That is, (17) transforms (15) into the following form of Lagrangian in the $\tau\xi$ coordinate frame,

$$\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L = \frac{1}{2}\left(\left(\frac{d\tilde{\tau}}{dp}\right)^2 - \left(\frac{d\xi}{dp}\right)^2\right) \tag{18}$$

which indicates that particles experience no gravity in the frame and follow straight motion with constant speed. The proper distance in the freely-falling $\tau\xi$ frame is,

$$d\bar{s}^2 = d\tilde{\tau}^2 - d\xi^2. \tag{19}$$

However, the static homogeneous gravity can not be geometrized. That is, both the $\tilde{\tau}\xi$ frame and the $\tilde{t}x$ can not say they have a curved spacetime. The former indicates a flat spacetime because it detects no gravity. So does the latter because the two frames are connected by a global coordinate transformation (17). Global coordinate transformations can not change the flatness of spacetime. Consider an example: sphere surface. You can not transform its metric form into the Pythagoras theorem by whatever global coordinate transformation.

(iii) Lagrange's equation and its solution. The canonical momentums conjugate to time $\tilde{t}$ and space coordinate x are respectively,

$$\begin{aligned} P_0 &= \tfrac{\partial}{\partial \dot{\tilde{t}}} L = (1 - \tilde{g}^2\tilde{t}^2)\tfrac{d\tilde{t}}{dp} + \tilde{g}\tilde{t}\tfrac{dx}{dp}, \\ P_1 &= \tfrac{\partial}{\partial \dot{x}} L = \tilde{g}\tilde{t}\tfrac{d\tilde{t}}{dp} - \tfrac{dx}{dp} \end{aligned} \tag{20}$$

To find the Lagrange's equation, we need

$$\begin{aligned} &\tfrac{\partial}{\partial \tilde{t}} L = -\tilde{g}^2\tilde{t}\left(\tfrac{d\tilde{t}}{dp}\right)^2 + \tilde{g}\tfrac{d\tilde{t}}{dp}\tfrac{dx}{dp}, \\ &\tfrac{\partial}{\partial x} L = 0 \end{aligned} \tag{21}$$

which says x is a cyclic coordinate. Therefore, one component of the Lagrange,s equation is

$$P_1 = \text{constant} = \tilde{g}\tilde{t}\frac{d\tilde{t}}{dp} - \frac{dx}{dp} = V. \tag{22}$$

The temporal component is

$$\frac{d}{dp}\left(\frac{\partial}{\partial \dot{\tilde{t}}} L\right) - \frac{\partial}{\partial \tilde{t}} L = \frac{d^2\tilde{t}}{dp^2} = 0. \tag{23}$$

The solution of the last equation is

$$\frac{d\tilde{t}}{dp} = \text{constant} = W. \tag{24}$$

The equations (22) and (24) are our constants of motion. Their combination leads to the following straight motion with varying speed,

$$\frac{dx}{d\tilde{t}} = -\frac{V}{W} + \tilde{g}\tilde{t} \tag{25}$$

where both V and W are constants. However, this is not the full story. Since our Lagrangian is proportional to the effective distance $(d\bar{s}/dp)^2$ and we deal with causal motion only, we always have $d\bar{s}^2 \geq 0$. Substitution of our constants of motion into the Lagrangian (15) gives

$$\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L = \frac{1}{2}(W^2 - V^2) = \text{constant}. \tag{26}$$

This supports our previous assertion

$$\bar{s} \propto p. \tag{27}$$

Causal motion implies that we always have

$$|W| \geq |V|. \tag{28}$$

(iv) Varying light speed in static homogeneous gravitational field. Because $\tilde{g} = g/c^2 > 0$ and light has the maximum speed ($d\bar{s}^2 = 0$), we have $W = -V$ for the motion of light:

$$\frac{dx}{d\tilde{t}} = 1 + \tilde{g}\tilde{t}. \tag{29}$$

Therefore, light accelerates with time and its speed is always greater than its initial value

$$\frac{dx}{dt} \mid_{t=0} = c. \tag{30}$$

However, we see a difficulty in choosing the initial spatial position x_0 corresponding to the initial time $t = 0$. The difficulty results from choosing the time-dependent coefficients of the Lagrangian (15) which, however, describes spatial inhomogeneity. The Lagrangian obtained by spatial square coordinate-rescale given in He (2005c) does not have the difficulty.

(v) Galactic redshift if there were an anitropic universe of static homogeneous gravity. I will present a model of isotropic universe in the following Section where temporally inhomogeneous gravity exists. The large-scale universe is spatially homogeneous. That is, the universe is isotropic so that each observer sees the same in all directions. This is very strongly suggested by observation: the temperature of the microwave background radiation is independent of direction to one part in a thousand, according to a variety of experiments on various scales of angular resolution down to $1'$ (Berry, 1989). Therefore, it is impossible to have a model of the universe whose gravity has a specific spatial direction, i. e., the direction of x-axis. However, I give the following formula for us to be familiar with the calculation in the next Section,

$$z = \frac{\nu_1}{\nu_2} - 1 = \frac{\sqrt{g_{00}(\tilde{t}_1)}}{\sqrt{g_{00}(\tilde{t}_2)}} - 1 = \frac{\sqrt{1 - \tilde{g}^2 \tilde{t}_1^2}}{\sqrt{1 - \tilde{g}^2 \tilde{t}_2^2}} - 1. \tag{31}$$

This redshift formula can not be combined with the light travel formula $x = x_0 + \tilde{t} + \frac{1}{2}\tilde{g}\tilde{t}^2$ to give the known Hubble law.

3 A Flat-Universe Model Based on Simple and Consistent Principles

(i) A set of simple and basic principles about the large-scale universe. Our current knowledge of the universe is very limited and all models of the universe are mainly based on some assumptions. The only must-explained observational facts by any model are the Hubble law that all galactic spectral-line redshifts are proportional to corresponding galactic distances from us, and the ‘accelerated expansion‘. The standard expanding-universe model follows the curved-spacetime assumption of general relativity (GR) and galactic redshifts are accordingly believed to be the Doppler-effect of the assumed galactic recession. However, curved spacetime is a faulty assumption. A correct model of the universe must

be based on some absolute inertial frame (the flat background spacetime). The existence of such a frame is shown to be true from the following three basic principles: (1) the universe has an isotropic but temporally inhomogeneous gravitational field; (2) the gravity is described by a Lagrangian which is the generalization to the proper distance of special relativity (the metric form of GR); (3) Hubble law is approximately true. These lead to varying light speed with time and give account of galactic redshifts and Hubble law (including 'accelerated expansion') as demonstrated in the following.

(ii) Lagrangian, Lagrange's equation and its solution. The above set of principles uniquely determine the following Lagrangian which describes the isotropic gravitational field in the universe,

$$\begin{aligned} \tfrac{1}{2}\grave{s}^2 &= L(x^0, x^i, \grave{x}^0, \grave{x}^i) \\ &= \tfrac{1}{2}B(\tilde{t})(\grave{x}^0)^2 - \tfrac{1}{2}A(\tilde{t})\textstyle\sum_{i=1}^3(\grave{x}^i)^2 \\ &= \tfrac{1}{2}g_{\alpha\beta}\tfrac{dx^\alpha}{dp}\tfrac{dx^\beta}{dp} \end{aligned} \tag{32}$$

where

$$g_{00} = B(\tilde{t})(> 0), g_{11} = g_{22} = g_{33} = -A(\tilde{t})(< 0),\ g_{\alpha\beta} = 0(\alpha \neq \beta). \tag{33}$$

The canonical momentums conjugate to time $\tilde{t}$ and space coordinates x^i are respectively,

$$\begin{aligned} P_0 &= \tfrac{\partial}{\partial \grave{x}^0}L = B\tfrac{d\tilde{t}}{dp} \\ P_i &= \tfrac{\partial}{\partial \grave{x}^i}L = -A\tfrac{dx^i}{dp},\ i = 1,2,3. \end{aligned} \tag{34}$$

To find the Lagrange's equation, we need

$$\begin{aligned} &\tfrac{\partial}{\partial x^0}L = \tfrac{1}{2}B'\left(\tfrac{d\tilde{t}}{dp}\right)^2 - \tfrac{1}{2}A'\textstyle\sum_{i=1}^3\left(\tfrac{dx^i}{dp}\right)^2, \\ &\tfrac{\partial}{\partial x^i}L = 0,\ i = 1,2,3, \\ &\tfrac{d}{dp}\left(\tfrac{\partial}{\partial \grave{\tilde{t}}}L\right) = B'\left(\tfrac{d\tilde{t}}{dp}\right)^2 + B\tfrac{d^2\tilde{t}}{dp^2} \end{aligned} \tag{35}$$

where A' and B' are derivatives with time $\tilde{t}$:

$$A' = \frac{dA(\tilde{t})}{d\tilde{t}},\ B' = \frac{dB(\tilde{t})}{d\tilde{t}}. \tag{36}$$

The middle equation in (35) indicates that $x^i, i = 1,2,3$ are cyclic coordinates. Therefore, the spatial components of the Lagrange's equation are

$$P_i = \text{constant} = -A(\tilde{t})\frac{dx^i}{dp} =,\ i = 1,2,3. \tag{37}$$

The temporal component is

$$\frac{d}{dp}\left(\frac{\partial}{\partial \grave{\tilde{t}}}L\right) - \frac{\partial}{\partial \tilde{t}}L = B\frac{d^2\tilde{t}}{dp^2} + \frac{1}{2}B'\left(\frac{d\tilde{t}}{dp}\right)^2 + \frac{1}{2}A'\sum_{i=1}^3\left(\frac{dx^i}{dp}\right)^2 = 0. \tag{38}$$

Combination with (37) gives

$$B\frac{d^2\tilde{t}}{dp^2} + \frac{1}{2}B'\left(\frac{d\tilde{t}}{dp}\right)^2 + \frac{A'}{2A^2}((P_1)^2 + (P_2)^2 + (P_3)^2) = 0. \tag{39}$$

We define the constant (conservative) spatial momentum P of the particle (e. g., a galaxy or a light crest from a galaxy),

$$P^2 = (P_1)^2 + (P_2)^2 + (P_3)^2. \tag{40}$$

Finally the solution of the equation (39) is

$$\frac{dp}{d\tilde{t}} = \sqrt{\frac{A(\tilde{t})B(\tilde{t})}{P^2 + WA(\tilde{t})}}. \tag{41}$$

where W is another constant. Substitution of the solution to the spatial ones (37) we finally have the particle's motion in our universe

$$\frac{dx^i}{d\tilde{t}} = -P_i\sqrt{\frac{B(\tilde{t})}{(P^2 + WA(\tilde{t}))A(\tilde{t})}}. \tag{42}$$

However, this is not the full story. Since our Lagrangian is proportional to the effective distance $(d\bar{s}/dp)^2$ and we deal with causal motion only, we always have $d\bar{s}^2 \geq 0$. Substitution of all our solutions into (32) gives

$$\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L = \frac{1}{2}W = \text{constant}. \tag{43}$$

This supports our previous assertion

$$\bar{s} \propto p. \tag{44}$$

Causal motion implies that we always have

$$W \geq 0. \tag{45}$$

(iii) Varying light speed in the gravitational field of the universe. Because light has the maximum speed ($d\bar{s}^2 = 0$), we have $W = 0$ for the motion of light. In its propagation direction we have

$$\frac{dx}{d\tilde{t}} = \sqrt{\frac{B(\tilde{t})}{A(\tilde{t})}}. \tag{46}$$

Currently the universe is at the time of

$$\tilde{t} = \tilde{t}_1 = ct_1. \tag{47}$$

The current light speed is $c \simeq 3 \times 10^8 \mathrm{m\,s^{-1}}$ which is used in the definition of $\tilde{t}$: $\tilde{t} = ct$. It is not wrong that we choose other light speed for the definition.

(iv) Galactic redshift and Hubble law. Galactic redshift is the formula (12)

$$z = \frac{\nu_1}{\nu_2} - 1 = \frac{\sqrt{g_{00}(\tilde{t}_1)}}{\sqrt{g_{00}(\tilde{t}_2)}} - 1 = \frac{\sqrt{B(\tilde{t}_1)}}{\sqrt{B(\tilde{t}_2)}} - 1. \tag{48}$$

We see that $B(\tilde{t})$ must be a monotonously increasing function with time for us to have galactic redshifts rather than blueshifts,

$$B(\tilde{t}) \uparrow . \tag{49}$$

The distance D between the two galaxies 1 (Milky Way) and 2 is given by the integral of the light travel formula (46)

$$D = \int_{\tilde{t}_2}^{\tilde{t}_1} \frac{dx}{d\tilde{t}} d\tilde{t} = \int_{\tilde{t}_2}^{\tilde{t}_1} \sqrt{\frac{B(\tilde{t})}{A(\tilde{t})}} d\tilde{t}. \tag{50}$$

The distance formula must have a redshift factor to give the Hubble law. This indicates that $A(\tilde{t})$ depends on $B(\tilde{t})$. A simple and general model of the dependence is

$$A(\tilde{t}) = \frac{B^{m+1}(\tilde{t})}{N^2 B'^2(\tilde{t})} \tag{51}$$

where m is a constant and $N(> 0)$ is another constant whose unit is length. Finally we have Hubble law,

$$\begin{aligned} D &= \tfrac{2N}{m-2} \left(\frac{1}{\sqrt{B(\tilde{t}_2)}^{m-2}} - \frac{1}{\sqrt{B(\tilde{t}_1)}^{m-2}} \right) \\ &= \tfrac{2N}{m-2} \left(\frac{1}{\sqrt{B(\tilde{t}_2)}} - \frac{1}{\sqrt{B(\tilde{t}_1)}} \right) \left(\frac{1}{\sqrt{B(\tilde{t}_2)}^{m-3}} + \dots \right) \\ &= \frac{2Nz}{(m-2)\sqrt{B(\tilde{t}_1)}} \left(\frac{1}{\sqrt{B(\tilde{t}_2)}^{m-3}} + \dots \right) \\ &= \frac{cz}{H_0(\tilde{t}_2, \tilde{t}_1)} \end{aligned} \tag{52}$$

where the Hubble constant H_0 is

$$H_0 = \frac{c(m-2)\sqrt{B(\tilde{t}_1)}}{2N} / \left(\frac{1}{\sqrt{B(\tilde{t}_2)}^{m-3}} + \dots \right). \tag{53}$$

As a summary, I note that the redshift requires $B(\tilde{t})$ be a monotonously increasing function of time and Hubble law requires A be determined by the function B (see (51)). Therefore, the only one degree of freedom left is the function form of $B(\tilde{t})$.

(v) Accelerated Expanding Universe. If H_0 depended only on $\tilde{t}_1$, the current time, then Hubble law would be perfectly true. However, it depends on the past time of the galaxy we observe,

$$H_0 = H_0(\tilde{t}_2, \tilde{t}_1). \tag{54}$$

If we assume

$$m > 3$$

then Hubble constant H_0 is not constant and increases with the past time $\tilde{t}_2$, of which the galaxy is observed. This increase with time of H_0 is explained as the 'accelerating expansion' of the universe. However, in my model, spacetime is flat (no expansion of curved spacetime) and the redshift is gravitational one which results from the evolution of the universe (mass density varies with time). Because redshift requires increasing $B(\tilde{t})$, we see that 'accelerating expansion' is consistent to galactic redshift.

(vi) Infinite Light Speed and the Birth of the Universe. Positive and increasing quantity $B(\tilde{t})$ indicates a time $\tilde{t}_0$, when $B(\tilde{t}_0) = 0$. This is the starting time of the universe. We can choose $\tilde{t}_0 = 0$ to

be the time of birth. Currently we do not know the exact physics at the hot birth. One thing is sure that light speed at the time must be infinite. Only infinite speed of communication could result in a later spatially homogeneous mass distribution in the infinite flat universe. This resolves the horizon and flatness problems due to birth. Infinite initial light speed indicates a decrease of light speed with time. Observation during the last decade does support the result of decreasing light-speed with time. The formula of light speed is (46). Therefore, decreasing light speed imposes further condition on the evolving factor $B(\tilde{t})$,

$$2BB'' \leq mB'. \tag{55}$$

(vii) Light Speed Constancy and the Death of the Universe. However, there is strong evidence that light speed is approximately constant during mature stage of the universe. Constant light speed with time means that $A(\tilde{t})$ and $B(\tilde{t})$ are the same

$$A(\tilde{t}) \equiv B(\tilde{t}).$$

They serve as the scaling factor. Perfect Hubble Redshift-distance linear law completely determines the scaling factor,

$$\frac{1}{B(\tilde{t})} \equiv \frac{1}{A(\tilde{t})} = \frac{1}{B_0} - M(\tilde{t} - \tilde{t}_0) \tag{56}$$

where M is a constant and $B_0 = B(\tilde{t}_0)$. This formula indicates a finite time $\tilde{t}_1$ when $M(\tilde{t}_1 - \tilde{t}_0) = 1/B_0$. This is the ending time of the universe because the scaling factor reaches infinity. The possibility of a rebirth needs further investigation.

(viii) The Absolute Inertial Frame of the Universe. Our calculation and results are reference-frames depended. For example, photon frequency is dependent on reference frames. Our results are meaningful only when single preferred inertial frame of the universe exists and the results are calculated with respect to the frame. The absolute frame is meaningful only when all components (e.g., galaxies) of the universe have convergent motion with respect to the frame. That is, all components slow down their speed of motion with respect to the frame. Since the nineteenth century, scientific report on the evidences of absolute inertial frame has never been stopped. Because of light speed constancy we have $A(\tilde{t}) \equiv B(\tilde{t})$ in the formula (42). We can see that the absolute speed of material particles (galaxies) does decrease with time, slowing-down motion with respect to the absolute inertial frame (note that $W > 0$ for material particles). Here we see that the existence of absolute inertial frame is once again the direct result of galactic redshift.

(viiii) The Variance with Time of Matter Distribution in the Universe. Our Lagrangian is defined on flat spacetime and can be quantized according to the classical and covariant quantization procedure (He, 2006). Because the spatial distribution of matter in the universe is homogeneous, the resulting amplitude of the wave function must be proportional to the density of the distribution. Astronomical observation suggests that the density decreases with time especially during early universe. We can

see that the amplitude does decrease with time if $B(\tilde{t})$ increases with time. That is, the astronomic observation is once again consistent to the result of galactic redshift.

4 Conclusion

I studied isotropic gravity. Isotropic gravity is the basis of the flat universe model given in the present paper as well as the basis of standard expanding universe model. Any isotropic gravitational field is similar to fluid pressure which, at each spatial position, exerts the same amount of force in all directions. Therefore, static frame in the field remains static and the isotropic gravitational force is not completely embodied by particle's motion. Einstein's equivalence principle is nothing but the cancellation of gravitational force by the particle's motion under the force, which is not true to isotropic gravitational force. This is called isotropic non-equivalence which holds to isotropic gravity (e. g., the large-scale universe) of pure temporal inhomogeneity. Realistic gravity is non-static, which is the case between the static gravity of pure spatial inhomogeneity and the isotropic gravity of pure temporal inhomogeneity. This indicates that the assumption of curved spacetime is a fundamental mistake. Therefore, a correct theory of gravity or a model of the universe must be based on the absolute flat background spacetime. The existence of such absolute spacetime is shown to be true from the following three basic principles about the universe: (1) the universe evolves (that is, there is an isotropic but temporally inhomogeneous gravitational field); (2) the gravity is described by a Lagrangian which is the generalization to the proper distance of special relativity (the metric form of GR); (3) Hubble law is approximately true. These lead to varying light speed and give account of galactic redshifts and Hubble law and most observational facts. Therefore, the assumptions of universal equivalence principle, big bang, and expansion are incorrect.

Local galactic blueshift is observed which is the Doppler effect of galactic motion due to local spatial inhomogeneity, i. e, the inhomogeneous distribution of matter. Quasars' redshift should be partly due to gravity of spatial inhomogeneous mass distribution. The static gravity of pure spatial inhomogeneity is given detailed account by He (2005a, 2005b, 2005c, and 2006). The averaged spectral line shifts in the universe are due to the gravity of the large-scale universe which is isotropic but temporally inhomogeneous. How the local spatial inhomogeneity is connected to the large-scale temporal inhomogeneity is the subject of cosmologic dynamics and is left to be future work.

5 Appendix

5.1 Radial Free Fall Motion toward Earth

Now I present the main result of appendix Section. Based on Schwarzschild metric, the formula of acceleration of radially freely-falling body is the following, which is proportional to the body's energy per unit mass (called PUM energy, $\tilde{E}$),

$$a(r) = \frac{d^2r}{dt^2} = \frac{2r_g}{r^2}\left(1 - \frac{2r_g}{r}\right)\left(c^2 + 3\left(1 - \frac{2r_g}{r}\right)\tilde{E}\right) \tag{57}$$

where the constant $r_g = GM/c^2$ is called Schwarzschild radius, G is gravitational constant, and M is the central point mass. The above formula and the following ones will be derived in the next Section. For the earth, $r_g = 4.43 \times 10^{-3}$ m. Near the earth surface, the formula can be approximated as

$$a(r_0) = \frac{d^2r}{dt^2} = 2g + \frac{6g}{c^2}\tilde{E} \tag{58}$$

where r_0 is the earth radius and g is the absolute value of the familiar acceleration

$$g \approx 9.8\ \mathrm{m\,s^{-2}}. \tag{59}$$

Note that the definition of PUM energy $\tilde{E}$ in my formulation differs from the one, E, in Newtonian mechanics by a constant $c^2/2$,

$$\tilde{E} = E - \frac{1}{2}c^2. \tag{60}$$

I will show in the next Section that $\tilde{E}$ is non-positive,

$$\tilde{E} \le 0. \tag{61}$$

Zero PUM energy ($\tilde{E} = 0$) corresponds to the motion of light. Taking $\tilde{E} = 0$ in (57), we have the radial acceleration of light,

$$a_{\max} = \frac{d^2r}{dt^2} = 2\frac{r_g}{r^2}\left(1 - \frac{2r_g}{r}\right)c^2 > 0. \tag{62}$$

Because the radial acceleration is positive, light decelerates toward the central mass. Therefore, light suffers repulsive force from the mass, contrary to people's imagination. This result of light deceleration is verified by the radar-echo-delay experiments (Shapiro, 1968 and 1971) and other similar experiments. Taking earth as example, we have the maximum acceleration (62) near earth surface,

$$a_{\max} \approx 2g. \tag{63}$$

We will know that the radial light speed is

$$\frac{dr}{dt} = c\left(1 - \frac{2r_g}{r}\right), \tag{64}$$

which is less than the familiar constant in the absence of gravity, c. Now we determine the upper and lower acceleration limits for material particles at a fixed position r. The acceleration of light, (62), is the upper limit. The lower limit of PUM energy for material particles is

$$\tilde{E}_{\min} = -\frac{c^2}{2(1 - 2r_g/r)}. \tag{65}$$

Therefore, the lower acceleration limit for material particles is

$$a_{\min} = \frac{d^2 r}{dt^2} = -\frac{r_g}{r^2}\left(1 - \frac{2r_g}{r}\right)c^2 < 0 \tag{66}$$

which is negative and corresponds to positive acceleration to the mass center. Therefore, low PUM-energy bodies do suffer attracting force. This minimum acceleration near earth surface is

$$a_{\min} \approx -g \tag{67}$$

which is what Einstein thought to be the constant acceleration for all test particles of whatever energy near the earth surface.

The above formulas and all other related formulas about Schwarzschild solution will be derived in the next Sub-section. The last Sub-section is a summary.

5.2 Freely-falling Motion Based on Schwarzschild Solution

(i) The Lagrangian of gravity. Now I derive the formulas in the last Section. All the calculation is based on the following Schwarzschild metric,

$$\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L(x^0, x^i, \grave{x}^0, \grave{x}^i) = -\frac{1}{2}B(r)\grave{\tilde{t}}^2 + \frac{1}{2}A(r)\grave{r}^2 + \frac{1}{2}r^2(\grave{\theta}^2 + \sin^2\theta\grave{\phi}^2), \tag{68}$$

where

$$B(r) = 1 - \frac{2r_g}{r}, \quad A(r) = \frac{1}{B(r)} = \frac{1}{1 - 2r_g/r} \tag{69}$$

and

$$\grave{x}^\alpha = \frac{dx^\alpha}{dp}, \text{ etc.}, \tag{70}$$

p is the curve parameter of test particle's motion, $(x^0 = ct = \tilde{t}, x^1 = x, x^2 = y, x^3 = z)$ is the rectangular coordinates on the spacetime, c is light speed. Note that, from now on, any letter with grave accent (e. g., $\grave{x}$) denotes the derivative of the quantity with the curve parameter p. This is to be distinguished from the dot accent (e. g., $\dot{x}$) which is the derivative of the quantity with time t, the common notation.

I assume that (68) is Lagrangian. Test particle follows the corresponding Lagrange's equation. According to GR, (68) is metric form (the half squared derivative of the proper distance $\bar{s}$), and test particle follows its geodesic equation. However, the two equations are identical because they are solutions of extremizing two related quantities respectively, the proper distance and the Lagrangian.

Because $A(r)$ and $B(r)$ depend only on polar distance r, the angular momentum of test particle is conserved and its spatial motion is planar. For simplicity, I choose $\theta = \pi/2$ in (68),

$$\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L(x^0, x^i, \grave{x}^0, \grave{x}^i) = -\frac{1}{2}B(r)\grave{\tilde{t}}^2 + \frac{1}{2}A(r)\grave{r}^2 + \frac{1}{2}r^2\grave{\phi}^2. \tag{71}$$

The Lagrangians (68) and (71) are negative so that they describe causal motion of material particles. They can be zero and describe the motion of light. According to the formulation, light speed in the presence of gravity is no longer the constant c but varies with position on spacetime. However, it is still the maximum one at each position.

(ii) The canonical momentums and the Lagrange's equation. The canonical momentums conjugate to time $\tilde{t}$, r, and ϕ are respectively,

$$\begin{aligned} P_0 &= \tfrac{\partial}{\partial \dot{\tilde{x}}^0} L = -B \tfrac{d\tilde{t}}{dp}, \\ P_r &= \tfrac{\partial}{\partial \dot{r}} L = A \tfrac{dr}{dp}, \\ P_\phi &= \tfrac{\partial}{\partial \dot{\phi}} L = r^2 \tfrac{d\phi}{dp} \end{aligned} \tag{72}$$

To find the Lagrange's equation, we do the following calculation

$$\begin{aligned} &\tfrac{\partial}{\partial x^0} L = 0, \\ &\tfrac{\partial}{\partial r} L = -\tfrac{1}{2} B' \left(\tfrac{d\tilde{t}}{dp}\right)^2 + \tfrac{1}{2} A' \left(\tfrac{dr}{dp}\right)^2 + r \left(\tfrac{d\phi}{dp}\right)^2, \\ &\tfrac{\partial}{\partial \phi} L = 0, \\ &\tfrac{d}{dp}\left(\tfrac{\partial}{\partial \dot{r}} L\right) = A' \left(\tfrac{dr}{dp}\right)^2 + A \tfrac{d^2 r}{dp^2} \end{aligned} \tag{73}$$

where A' and B' are derivatives with polar distance r:

$$A' = \frac{dA(r)}{dr}, \;\; B' = \frac{dB(r)}{dr}. \tag{74}$$

The formulas (73) indicate that $\tilde{t}, \phi$ are cyclic coordinates. Therefore, the temporal component and the angle component of the Lagrange's equation are

$$\begin{aligned} P_0 &= \text{constant} = -B(r) \tfrac{d\tilde{t}}{dp} = -c, \\ P_\phi &= \text{constant} = r^2 \tfrac{d\phi}{dp} = J. \end{aligned} \tag{75}$$

For simplicity, I have chosen the first constant to be $-c$. The polar distance component is

$$\frac{d}{dp}\left(\frac{\partial}{\partial \dot{r}} L\right) - \frac{\partial}{\partial r} L = A \frac{d^2 r}{dp^2} + \frac{1}{2} A' \left(\frac{dr}{dp}\right)^2 + \frac{1}{2} B' \left(\frac{d\tilde{t}}{dp}\right)^2 - r \left(\frac{d\phi}{dp}\right)^2 = 0. \tag{76}$$

The equation can be intFGRated with the help of (75) and finally we have all constants of motion which define the particle's motion of freely falling,

$$\frac{dt}{dp} = \frac{1}{B(r)}, \tag{77}$$

$$r^2 \frac{d\phi}{dp} = J, \tag{78}$$

$$\frac{1}{2}\left(\frac{A(r)}{B^2(r)}\left(\frac{dr}{dt}\right)^2 + \frac{J^2}{r^2} - \frac{c^2}{B(r)}\right) = \tilde{E}\,(\text{constant}). \tag{79}$$

In the next part I will prove that the constant $\tilde{E}$ is the energy per unit mass (PUM energy). The formulas (77), (78) and (79) are identical to the ones of geodesic equation derived in GR. They must be identical because they are solutions of extremizing two related quantities respectively, the proper distance and the Lagrangian.

(iii) Energy and covariant Hamiltonian. To determine the PUM energy associated with the freely falling body, we need to find its Hamiltonian H,

$$\begin{aligned} &H(x^0, x^i, P_0, P_i) = \grave{x}^0 P_0 + \grave{r} P_r + \grave{\phi} P_\phi - L(x^0, x^i, \grave{x}^0, \grave{x}^i) \\ &= -\tfrac{1}{2} B(r) \grave{t}^2 + \tfrac{1}{2} A(r) \grave{r}^2 + \tfrac{1}{2} r^2 \grave{\phi}^2 \equiv L(x^0, x^i, \grave{x}^0, \grave{x}^i). \end{aligned} \tag{80}$$

Substituting the constants of motion into the Hamiltonian, we have

$$H = \tilde{E} = -\frac{1}{2}\left(\frac{d\bar{s}}{dp}\right)^2 = L \tag{81}$$

which indicates that $\tilde{E}$ is our energy (PUM energy). We also see that the temporal part of the Hamiltonian is potential energy while the spatial one is kinetic energy. Because L describes causal motion, both L and $\tilde{E}$ are non-positive, which verifies the formula (61). Therefore, the upper limit of the PUM energy is

$$\tilde{E}_{\max} = 0. \tag{82}$$

The Newtonian approximation of the formula (79) is

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 + \frac{J^2}{2r^2} - \frac{GM}{r} - \frac{c^2}{2} = E - \frac{c^2}{2} = \tilde{E}. \tag{83}$$

Therefore, the PUM energy $\tilde{E}$ in my formulation differs from the one, E, in Newtonian mechanics by a constant $c^2/2$ (see the formula (60)).

(iv) The acceleration of the radially freely falling bodies. Freely falling body is described by the equation (79) which can be changed into

$$\left(\frac{dr}{dt}\right)^2 + \frac{J^2 B^3(r)}{r^2} - c^2 B^2(r) = 2\tilde{E} B^3(r). \tag{84}$$

For simplicity, I consider the freely-falling bodies which have only radial motion to the central mass. That is, they do not have angular momentum J. Choosing $J = 0$ and $dr/d\tilde{t} = 0$ in (79), we have the lower limit of the PUM energy (65). Choosing $J = 0$ and taking derivatives with t on the two sides of (84), we have the formula of acceleration,

$$a(r) = \frac{d^2 r}{dt^2} = \frac{2r_g}{r^2} B(r)(c^2 + 3B(r)\tilde{E}) \tag{85}$$

which is exactly the formula (57). Choosing $J = 0$ and $\tilde{E} = 0$ in (79), we have the light speed (64). All formulas about Schwarzschild solution in the last Section are verified.

5.3 Summary

Based on the Schwarzschild solution of general relativity, the acceleration of freely-falling bodies in the gravitational field due to single isolated point mass is studied. For simplicity, I consider radially freely-falling bodies. At each spatial position, the acceleration depends linearly on the body's energy per unit mass, and low energy bodies accelerate toward the central mass (suffer attracting forces) while high energy bodies decelerate (suffer repulsive forces, contrary to people's imagination). Photons have the maximum speeds and, therefore, suffer repulsive forces. This result is verified by the standard radar-echo-delay experiments (Shapiro, 1968 and 1971).

Take earth as example. Near the earth surface, the correct formula of acceleration can be approximated as

$$a(r_0) = \frac{d^2r}{dt^2} = 2g + \frac{6g}{c^2}\tilde{E} \tag{86}$$

where r_0 is the earth radius, g is the familiar acceleration $g \approx 9.8\ \mathrm{m\,s^{-2}}$, and the PUM (per unit mass) energy $\tilde{E}$ differs from the one, E, in Newtonian mechanics by a constant $c^2/2$ (see (60)). The upper limits of $\tilde{E}$ and a are zero and $2g$ respectively, which can be reached only by light. Therefore, light and high-energy material particles suffer repulsive forces from the central mass (earth). The lower limits are $-c^2/2$ and $-g$ respectively. Therefore, low-energy material particles do suffer attracting forces. Our results question Einstein's equivalence principle because locally we can not choose single observational frame which accelerates and decelerates simultaneously. The formula (86) indicates that we do not suffer gravity if our energy per unit mass reaches minus one third of squared light-speed, i. e. one sixth of squared light-speed in terms of Newtonian mechanics energy.

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